

PLASMA WAVES REFLECTION FROM A BOUNDARY WITH SPECULAR ACCOMMODATIVE BOUNDARY CONDITIONS

N. V. Gritsienko, A. V. Latyshev, A. A. Yushkanov

Moscow State Regional University, Radio st., 10a, Moscow 105005, Russia

e-mail: natafmf@yandex.ru, avlatyshev@mail.ru, yushkanov@inbox.ru

Abstract

In the present work the linearized problem of plasma wave reflection from a boundary of a half-space is solved analytically. Specular accommodative conditions of plasma wave reflection from plasma boundary are taken into consideration. Wave reflectance is found as function of the given parameters of the problem, and its dependence on the normal electron momentum accommodation coefficient is shown by the authors. The case of resonance when the frequency of self-consistent electric field oscillations is close to the proper (Langmuir) plasma oscillations frequency, namely, the case of long wave limit is analyzed in the present paper. Refs. 17. Figs. 6.

Keywords: degenerate plasma, half-space, normal electron momentum accommodation coefficient, specular accommodative boundary condition, long wave limit, wave reflectance.

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1. BASIC EQUATIONS

Research of degenerate electron plasma behaviour, processes which take place in plasma under electric field, plasma waves becomes more and more actual at the present time in connection with the problems of such intensively developing areas as microelectronics and nanotechnologies [1] – [6].

This work continues the research of the electron plasma behaviour in external longitudinal alternating electric field [7] – [11].

In the present work linearized problem of plasma wave reflection from a boundary of a half-space of conductive medium is solved analytically. Specular accommodative conditions of electron reflection from plasma boundary are taken into consideration. The diffuse boundary conditions were considered in [8]–[10].

Expression for wave reflectance is obtained and it is shown that in the case when normal electron momentum accommodation coefficient takes on a value of zero the wave reflectance is expressed by the formula obtained earlier in [8], [11].

Let us consider degenerate plasma which is situated in a half-space $x > 0$. We assume that self-consistent electric field $\mathbf{E}(\mathbf{r}, t)$ inside plasma has one x -component and varies along the axis x only: $\mathbf{E} = \{E_x(x, t), 0, 0\}$.

In this case the electric field is perpendicular to the plasma boundary which is situated in the plane $x = 0$.

Here ω_p is Langmuir (proper) plasma oscillation frequency, $\omega_p = \frac{4\pi e^2 N}{m}$, N is the electron numerical density (concentration), m is the mass of the electron.

Let us take the system of equations which describes plasma behaviour. As the kinetic equation we take τ -model Vlasov — Boltzmann kinetic equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E} \frac{\partial f}{\partial \mathbf{p}} = \frac{f_{eq}(\mathbf{r}, t) - f(\mathbf{r}, \mathbf{v}, t)}{\tau}. \quad (1.1)$$

Here $f = f(\mathbf{r}, \mathbf{v}, t)$ is the electron distribution function, e is the electron charge, $\mathbf{p} = m\mathbf{v}$ is the electron momentum, m is the electron mass, τ is the characteristic time period between two collisions, $f_{eq} = f_{eq}(\mathbf{r}, t)$ is the local equilibrium distribution function of Fermi and Dirac, $f_{eq} = \Theta(\mathcal{E}_F(t, x) - \mathcal{E})$, where $\Theta(x)$ is the function of Heaviside,

$$\Theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases}$$

$\mathcal{E}_F(t, x) = \frac{1}{2}mv_F^2(t, x)$ is the disturbed kinetic energy of Fermi, $\mathcal{E} = \frac{1}{2}mv^2$ is the kinetic energy of the electron.

Let us consider the Maxwell equation for the electric field

$$\text{div } \mathbf{E}(\mathbf{r}, t) = 4\pi e \int (f(\mathbf{r}, \mathbf{v}, t) - f_0(\mathbf{v})) d\Omega_F, \quad (1.2)$$

where

$$d\Omega_F = \frac{2d^3p}{(2\pi\hbar)^3}, \quad d^3p = dp_x dp_y dp_z.$$

Here f_0 is the undisturbed electron distribution function of Fermi and Dirac, $f_0(\mathcal{E}) = \Theta(\mathcal{E}_F - \mathcal{E})$, \hbar is the Planck constant, $\mathcal{E}_F = \frac{1}{2}mv_F^2$ is the

undisturbed kinetic energy of Fermi, v_F is the electron velocity on the Fermi surface which is considered as spherical.

Let us search for the solution of the system (1.1) and (1.2) in the following form

$$f = f_0(\mathcal{E}) + \mathcal{E}_F \delta(\mathcal{E}_F - \mathcal{E}) H(x, \mu, t), \quad \mu = \frac{v_x}{v}.$$

We obtain (see [9]) linearized system of equations of Vlasov — Maxwell:

$$\frac{\partial H}{\partial t_1} + \mu \frac{\partial H}{\partial x_1} + H(x_1, \mu, t_1) = \mu e(x_1, t_1) + \frac{1}{2} \int_{-1}^1 H(x_1, \mu', t_1) d\mu', \quad (1.3)$$

$$\frac{\partial e(x_1, t_1)}{\partial x_1} = \frac{3\omega_p^2}{2\nu^2} \int_{-1}^1 H(x_1, \mu', t_1) d\mu'. \quad (1.4)$$

Here $e(x_1, t_1)$ is dimensionless function

$$e(x, t) = \frac{ev_F}{\nu \mathcal{E}_f} E_x(x, t),$$

$x_1 = x/l$ is the dimensionless coordinate, where $l = v_F \tau$ is the average free path of electrons, $t_1 = \nu t$ is the dimensionless time, ν is the effective frequency of electron scattering, $\nu = 1/\tau$.

Supposing that k is the dimensional wave number, we introduce the dimensionless wave number $k_1 = k \frac{v_F}{\omega_p}$, then we have $kx = \frac{k_1 x_1}{\varepsilon}$, where $\varepsilon = \frac{\nu}{\omega_p}$. Let us introduce the quantity $\omega_1 = \omega \tau = \omega/\nu$.

2. BOUNDARY CONDITIONS STATEMENT

Let the plasma wave move to the plasma boundary situated in the plane $x_1 = 0$. The electric field of the wave changes according to the following law

$$e_+(x_1, t_1) = E_1 \exp(-i(\frac{k_1 x_1}{\varepsilon} + \omega_1 t_1)). \quad (2.1)$$

The amplitude of this wave E_1 we assume to be given. On the plasma boundary this wave reflects and the electric field of the reflected wave has the following form

$$e_-(x_1, t_1) = E_2 \exp(i(\frac{k_1 x_1}{\varepsilon} - \omega_1 t_1)). \quad (2.2)$$

The amplitude E_2 is unknown and is to be found from the problem solution. The quantities ω_1 and k_1 are not independent, the following dependence $\omega_1 = \omega_1(k_1)$ is determined from the solution of the dispersion equation which will be introduced below.

It is required to determine what part of the wave energy is absorbed under the wave reflection from the plasma boundary, and what part of the energy is reflected, and also to find the phase shift of the wave. It means we have to calculate the reflectance which is determined as square of module of the ratio of reflected and incoming waves amplitudes

$$R(k, \omega, \varepsilon) = \left| \frac{E_2}{E_1} \right|^2 \quad (2.3)$$

and to find the argument of the amplitudes ratio

$$\phi(k, \omega, \varepsilon) = \arg \left(\frac{E_2}{E_1} \right) = \arg E_2 - \arg E_1. \quad (2.4)$$

Let us outline the time variable of the functions $H(x_1, \mu, t_1)$ and $e(x_1, t_1)$, assuming

$$H(x_1, \mu, t_1) = e^{-i\omega_1 t_1} h(x_1, \mu), \quad e(x_1, t_1) = e^{-i\omega_1 t_1} e(x_1). \quad (2.5)$$

The system of equations (1.3) and (1.4) in this case will be transformed to the following form:

$$\mu \frac{\partial h}{\partial x_1} + (1 - i\omega_1) h(x_1, \mu) = \mu e(x_1) + \frac{1}{2} \int_{-1}^1 h(x_1, \mu') d\mu', \quad (2.6)$$

$$\frac{de(x_1)}{dx_1} = \frac{3\omega_p^2}{2\nu^2} \int_{-1}^1 h(x_1, \mu') d\mu'. \quad (2.7)$$

Further instead of x_1, t_1 we write x, t . We rewrite the system of equations (2.6) and (2.7) in the form:

$$\mu \frac{\partial h}{\partial x} + z_0 h(x, \mu) = \mu e(x) + \frac{1}{2} \int_{-1}^1 h(x, \mu') d\mu', \quad z_0 = 1 - i\omega_1. \quad (2.8)$$

$$\frac{de(x)}{dx} = \frac{3}{2\varepsilon^2} \int_{-1}^1 h(x, \mu') d\mu'. \quad (2.9)$$

We consider the external electric field outside the plasma limit is absent. This means that for the field inside plasma on the plasma boundary the following condition is satisfied:

$$e(0) = 0. \quad (2.10)$$

The non-flowing condition for the particle (electric current) flow through the plasma boundary means that

$$\int_{-1}^1 \mu h(0, \mu) d\mu = 0. \quad (2.11)$$

In the kinetic theory for the description of the surface properties the accommodation coefficients are used often. Tangential momentum and energy accommodation coefficients are the most-used. For the problem considered the normal electron momentum accommodation under the scattering on the surface has the most important significance.

The normal momentum accommodation coefficient is defined by the following relation:

$$\alpha_p = \frac{P_i - P_r}{P_i - P_s}, \quad 0 \leq \alpha_p \leq 1, \quad (2.12)$$

where P_i and P_r are the flows of normal to the surface momentum of incoming to the boundary and reflected from it electrons,

$$P_i = \int_{-1}^0 \mu^2 h(0, \mu) d\mu, \quad P_r = \int_0^1 \mu^2 h(0, \mu) d\mu, \quad (2.13)$$

quantity P_s is the normal momentum flow for electrons reflected from the surface which are in thermodynamic equilibrium with the wall,

$$P_s = \int_0^1 \mu^2 h_s(\mu) d\mu, \quad \text{где} \quad h_s(\mu) = A_s, \quad 0 < \mu < 1. \quad (2.14)$$

The function $h_s(\mu)$ is the equilibrium distribution function of the corresponding electrons. This function is to satisfy the condition similar to the non-flowing condition:

$$\int_{-1}^0 \mu h(0, \mu) d\mu + \int_0^1 \mu h_s(\mu) d\mu = 0. \quad (2.15)$$

We are going to consider the relation between the normal momentum accommodation coefficient α_p and the diffuseness coefficient q for the case of specular and diffuse boundary conditions which are written in the following form:

$$h(0, \mu) = (1 - q)h(0, -\mu) + a_s, \quad 0 < \mu < 1.$$

Here q is the diffuseness coefficient ($0 \leq q \leq 1$), a_s is the quantity determined from the non-flowing condition.

From the non-flowing condition we derive

$$a_s = -2q \int_{-1}^0 \mu h(0, \mu) d\mu = qA_s.$$

After that we find the difference between the flows

$$\begin{aligned} P_i - P_r &= q \int_{-1}^0 \mu^2 h(0, \mu) d\mu - \int_0^1 \mu^2 a_s d\mu = \\ &= q \int_{-1}^0 \mu^2 h(0, \mu) d\mu - q \int_0^1 \mu^2 A_s d\mu = qP_i - qP_s. \end{aligned}$$

Substituting the expressions obtained to the definition of the normal momentum accommodation coefficient we obtain that $\alpha_p = q$.

Thus, for the specular and diffuse boundary conditions the normal momentum accommodation coefficient α_p coincides with the diffusion coefficient q .

Together with the specular and diffuse boundary conditions other variants of boundary conditions are used in the kinetic theory.

In particular, accommodative boundary conditions are widely used. They are divided into two forms: diffuse accommodative and specular accommodative boundary conditions (see [12]).

We consider specular accommodative boundary conditions. For the function h these conditions will be written in the following form:

$$h(0, \mu) = h(0, -\mu) + A_1 + A_2\mu, \quad 0 < \mu < 1. \quad (2.16)$$

Coefficients A_1 and A_2 can be derived from the non-flowing condition and the definition of the normal electron momentum accommodation coefficient.

The problem statement is completed. Now the problem consists in finding of such solution of the system of equations (2.8) and (2.9), which satisfies the boundary conditions (2.10)–(2.16). Further, with use of the amplitudes of reflected and incoming waves found it is required to find the reflectance of the incoming wave energy (2.3) and the argument of the amplitudes ratio (2.4).

3. THE RELATION BETWEEN FLOWS AND BOUNDARY CONDITIONS

First of all let us find expression which relates the constants A_0 , A_1 from the boundary condition (2.13).

To carry this out we will use the condition of non-flowing (2.12) of the particle flow through the plasma boundary, which we will write as a sum of two flows:

$$N_0 \equiv \int_0^1 \mu h(0, \mu) d\mu + \int_{-1}^0 \mu h(0, \mu) d\mu = 0.$$

After evident substitution of the variable in the second integral we obtain:

$$N_0 \equiv \int_0^1 \mu [h(0, \mu) - h(0, -\mu)] d\mu = 0.$$

Taking into account the relation (2.16), we obtain that $A_0 = -2A_1/3$. With the help of this relation we can rewrite the condition (2.16) in the following form:

$$h(0, \mu) = h(0, -\mu) + A_1(\mu - \frac{2}{3}), \quad 0 < \mu < 1. \quad (3.1)$$

We consider the momentum flow of the electrons which are moving to the boundary. According to (3.1) we have:

$$P_i = P_r - \frac{1}{36}A_1. \quad (3.2)$$

It is easy to see further that

$$P_s = A_s/3. \quad (3.3)$$

With the help of the formula (3.2) we will rewrite the definition of the accommodation coefficient (2.12) in the form:

$$\alpha_p P_r - \alpha_p \frac{A_s}{3} + \frac{A_1}{36}(1 - \alpha_p) = 0. \quad (3.4)$$

Let us consider the condition (2.15). From this condition we obtain that

$$A_s = -2 \int_{-1}^0 \mu h(0, \mu) d\mu = 2 \int_0^1 \mu h(0, -\mu) d\mu.$$

Using the condition (3.1), we then get

$$A_s = 2 \int_0^1 \mu h(0, \mu) d\mu. \quad (3.5)$$

Now with the help of the second equality from (2.13) and (3.4) we rewrite the relation (3.3) in the integral form:

$$\alpha_p \int_0^1 \left(\mu^2 - \frac{2}{3} \right) h(0, \mu) d\mu = -\frac{1}{36}(1 - \alpha_p) A_1. \quad (3.6)$$

Now the boundary problem consists of the equations (2.8) and (2.9) and boundary conditions (2.10), (3.1) and (3.6).

4. SEPARATION OF VARIABLES AND CHARACTERISTICAL SYSTEM

Application of the general Fourier method of the separation of variables in several steps results in the following substitution:

$$h_\eta(x, \mu) = \exp\left(-\frac{z_0 x}{\eta}\right) \Phi(\eta, \mu), \quad e_\eta(x) = \exp\left(-\frac{z_0 x}{\eta}\right) E(\eta), \quad (4.1)$$

where η is the spectrum parameter or the parameter of separation, which is complex in general.

We substitute the equalities (4.1) into the equations (2.8) and (2.9). We obtain the following characteristic system of equations:

$$z_0(\eta - \mu) \Phi(\eta, \mu) = \eta \mu E(\eta) + \frac{\eta}{2} \int_{-1}^1 \Phi(\eta, \mu') d\mu', \quad (4.2)$$

$$z_0 E(\eta) = -\frac{3}{\varepsilon^2} \cdot \frac{\eta}{2} \int_{-1}^1 \Phi(\eta, \mu') d\mu'. \quad (4.3)$$

Let us introduce the designations:

$$\gamma = \frac{\omega}{\omega_p} - 1, \quad \eta_1^2 = \frac{\varepsilon^2 z_0}{3}, \quad z_0 = 1 - i \frac{1 + \gamma}{\varepsilon}, \quad c = 2\eta_1^2 z_0.$$

Substituting the integral from the equation (4.3) into (4.2), we come to the following system of equations:

$$(\eta - \mu) \Phi(\eta, \mu) = \frac{E(\eta)}{z_0} (\eta \mu - \eta_1^2), \quad -\eta_1^2 E(\eta) = \frac{\eta}{2} \int_{-1}^1 \Phi(\eta, \mu') d\mu'. \quad (4.4)$$

Solution of the system (4.4) depends essentially on the condition if the spectrum parameter η belongs to the interval $-1 < \eta < 1$. In connection with this the interval $-1 < \eta < 1$ we will call as continuous spectrum of the characteristic system.

Let the parameter $\eta \in (-1, 1)$. Then from the equations (4.4) in the class of general functions we will find eigenfunction corresponding to the continuous spectrum:

$$\Phi(\eta, \mu) = F(\eta, \mu) \frac{E(\eta)}{z_0}, \quad (4.5)$$

where

$$F(\eta, \mu) = P \frac{\mu \eta - \eta_1^2}{\eta - \mu} - c \frac{\lambda(\eta)}{\eta} \delta(\eta - \mu). \quad (4.6)$$

In the equation (4.6) $\delta(x)$ is the delta-function of Dirac, the symbol Px^{-1} means the principal value of the integral under integrating of the expression x^{-1} , the fuction $\lambda(z)$ is called as dispersion function of the problem,

$$\lambda(z) = 1 + \frac{z}{c} \int_{-1}^1 \frac{\eta_1^2 - z\mu}{\mu - z} d\mu. \quad (4.7)$$

The function (4.5) is called eigenfunction of the continuous spectrum, since the spectrum parameter η fills out the continuum $(-1, +1)$ compactly. The eigensolutions of the given problem can be found from the equalities

(4.7). The dispersion function $\lambda(z)$ we express in the terms of the Case [13] dispersion function:

$$\lambda(z) = 1 - \frac{1}{z_0} + \frac{1}{z_0} \left(1 - \frac{z^2}{\eta_1^2}\right) \lambda_c(z),$$

where

$$\lambda_c(z) = 1 + \frac{z}{2} \int_{-1}^1 \frac{d\tau}{\tau - z} = \frac{1}{2} \int_{-1}^1 \frac{\tau d\tau}{\tau - z}$$

is the Case dispersion function [13].

The boundary values of the dispersion function from above and below the contour are calculated according to the Sokhotsky formulas

$$\lambda^\pm(\mu) = \lambda(\mu) \pm \frac{i\pi\mu}{2\eta_1^2 z_0} (\eta_1^2 - \mu^2), \quad -1 < \mu < 1,$$

from where we have

$$\begin{aligned} \lambda^+(\mu) - \lambda^-(\mu) &= \frac{i\pi}{\eta_1^2 z_0} \mu (\eta_1^2 - \mu^2), \\ \frac{\lambda^+(\mu) + \lambda^-(\mu)}{2} &= \lambda(\mu), \quad -1 < \mu < 1, \end{aligned}$$

where

$$\lambda(\mu) = 1 + \frac{\mu}{2\eta_1^2 z_0} \int_{-1}^1 \frac{\eta_1^2 - \eta^2}{\eta - \mu} d\eta,$$

and the integral in this equality is understood as singular in terms of the principal value by Cauchy. Besides that, the function $\lambda(\mu)$ can be represented in the following form:

$$\lambda(\mu) = 1 - \frac{1}{z_0} + \frac{1}{z_0} \left(1 - \frac{\mu^2}{\eta_1^2}\right) \lambda_c(\mu), \quad \lambda_c(\mu) = 1 + \frac{\mu}{2} \ln \frac{1 - \mu}{1 + \mu}.$$

5. EIGENFUNCTIONS OF THE DISCRETE SPECTRUM AND PLASMA WAVES

According to the definition, the discrete spectrum of the characteristic equation is a set of zeroes of the dispersion equation

$$\lambda(z)/z = 0. \quad (5.1)$$

We start to search zeroes of this equation. Let us expand take Laurent series of the dispersion function:

$$\lambda(z) = \lambda_\infty + \frac{\lambda_2}{z^2} + \frac{\lambda_4}{z^4} + \dots, \quad |z| > 1. \quad (5.1)$$

Here

$$\begin{aligned} \lambda_\infty &\equiv \lambda(\infty) = 1 - \frac{1}{z_0} + \frac{1}{3z_0\eta_1^2} = \frac{2\gamma + i\varepsilon + \gamma(\gamma + i\varepsilon)}{(1 + \gamma + i\varepsilon)^2}, \\ \lambda_2 &= -\frac{1}{z_0} \left(\frac{1}{3} - \frac{1}{5\eta_1^2} \right) = -\frac{9 + 5i\varepsilon(1 + \gamma + i\varepsilon)}{15(1 + \gamma + i\varepsilon)^2}, \\ \lambda_4 &= -\frac{1}{z_0} \left(\frac{1}{5} - \frac{1}{7\eta_1^2} \right) = -\frac{15 + 7i\varepsilon(1 + \gamma + i\varepsilon)}{35(1 + \gamma + i\varepsilon)^2}. \end{aligned}$$

One can easily see that in collisional plasma (i.e. when $\varepsilon > 0$) the coefficient $\lambda_\infty \neq 0$. Consequently, the dispersion equation has infinity as a zero $\eta_i = \infty$, to which the discrete eigensolutions of the given system correspond: $h_\infty(x, \mu) = \mu/z_0$, $e_\infty(x) = 1$.

This solution is naturally called as mode of Drude. It describes the volume conductivity of metal, considered by Drude (see, for example, [14]).

Let us consider the question of the plasma mode existence in details. We find finite complex zeroes of the dispersion function. We use the principle of argument. We take the contour $\Gamma_\varepsilon^+ = \Gamma_R \cup \gamma_\varepsilon$, (see Fig. 1), which is passed in the positive direction and which bounds the biconnected domain D_R . This contour consists of the circumference $\{\Gamma_R : |z| = R, R = 1/\varepsilon, \varepsilon > 0\}$, and the contour γ_ε , which includes the cut $[-1, +1]$, and stands at the distance of ε from it.

According to the principle of argument the number [15] of zeroes N in the area D_ε equals to:

$$N = \frac{1}{2\pi i} \oint_{\Gamma_\varepsilon} d \ln \lambda(z).$$

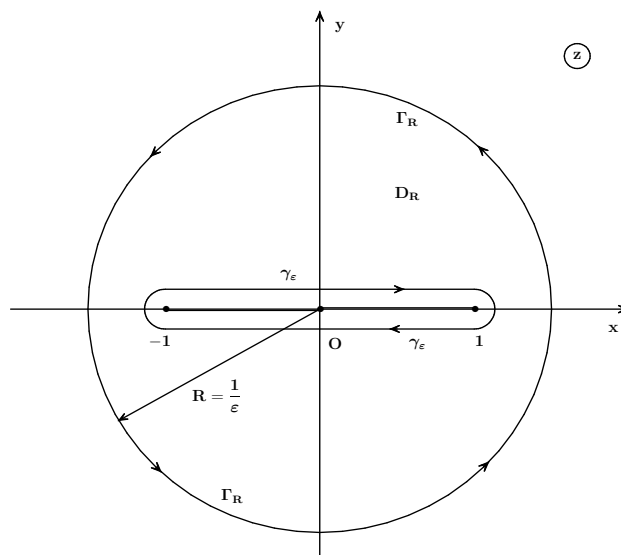


Fig. 1.

Considering the limit in this equality when $\varepsilon \rightarrow 0$ and taking into account that the dispersion function is analytic in the neighbourhood of the infinity, we obtain that

$$N = \frac{1}{2\pi i} \int_{-1}^1 d \ln \frac{\lambda^+(\tau)}{\lambda^-(\tau)} = \frac{1}{\pi i} \int_0^1 d \ln \frac{\lambda^+(\tau)}{\lambda^-(\tau)} = \frac{1}{\pi} \arg \frac{\lambda^+(\tau)}{\lambda^-(\tau)} \Big|_0^1. \quad (5.2)$$

Consider the curve

$$\gamma = \{z : z = G(\tau), 0 \leq \tau \leq +1\},$$

where $G(\tau) = \lambda^+(\tau)/\lambda^-(\tau)$. It is obvious that $G(0) = 1$, $\lim_{\tau \rightarrow 1} G(\tau) = 1$. Hence, according to (5.3), the number of zeroes N equals to the double number of turns of the curve γ round the point of origin, i.e. $N = 2\mathcal{N}(G)$, $\mathcal{N}(G) = \text{Ind}_{[0, +1]} G(\tau)$.

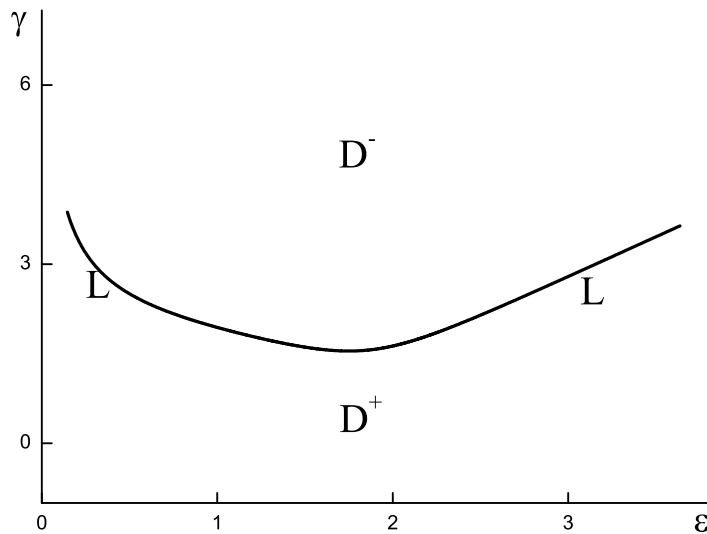


Fig. 2.

Thus, the number of zeroes of the dispersion function belonging to the complex plane out of the segment $[-1, 1]$ of the real axis equals to the double index of the function $G(\tau)$, calculated on the semi-axis $[0, +1]$.

We take on the plane (γ, ε) (see Fig. 2) the curve L , determined by equations: $\gamma = -1 + \sqrt{L_1(\tau)}$, $\varepsilon = \sqrt{L_2(\tau)}$, $0 \leq \tau \leq 1$, where

$$L_1(\tau) = -\frac{3\mu^2[s^2 + \lambda_0(1 + \lambda_0)]^2}{\lambda_0[s^2 + (1 + \lambda_0)^2]}, \quad L_2(\tau) = -\frac{3\tau^2 s^2}{\lambda_0[s^2 + (1 + \lambda_0)^2]}.$$

Here

$$\lambda_0(\tau) = 1 + \frac{\tau}{2} \ln \frac{1 - \tau}{1 + \tau}, \quad s(\tau) = \frac{\pi}{2} \tau.$$

In the same way, as was shown in the work [16], we can prove that if $(\gamma, \varepsilon) \in D^+$, then the index of the problem equals to unity, i.e. $N = 2$ is the number of zeroes which equals to two, and if $(\gamma, \varepsilon) \in D^-$, then the index of the problem equals to zero, i.e. $N = 0$.

We would like to notice, that in the work [17] (see also [16]) the method of examination of the boundary mode in the case when $(\gamma, \varepsilon) \in L$ was developed.

Since the dispersion function is even its zeroes differ from each other by sign. We designate these zeroes as following $\pm\eta_0$, by η_0 we take the zero which

satisfies the condition $\text{Re } \eta_0 > 0$. The following solution corresponds to the zero η_0 :

$$h_{\eta_0}(x, \mu) = \exp\left(-\frac{z_0 x}{\eta_0}\right) \frac{E_2}{z_0} \frac{\eta_0 \mu - \eta_1^2}{\eta_0 - \mu}, \quad e_{\eta_0}(x) = \exp\left(-\frac{z_0 x}{\eta_0}\right) E_2. \quad (5.2)$$

This solution is naturally called as mode of Debay (this is plasma mode). In the case of low frequencies it describes well-known screening of Debay [3]. The external field penetrates into plasma on the depth of r_D , r_D is the radius of Debay. When the external field frequencies are close to Langmuir frequencies, the mode of Debay describes plasma oscillations (see, for instance, [3, 14]).

From the equalities (2.2) for the wave $e_-(x, t)$ with the help of (2.6) and the equality (4.1) follows the relation between the wave number k and the zero of the dispersion function $\eta_0(\omega, \nu)$: $i \frac{kx}{\varepsilon} = -\frac{z_0 x}{\eta_0}$, hence $\eta_0 \equiv \eta_0(\gamma, \varepsilon) = \frac{1 + \gamma}{k} + i \frac{\varepsilon}{k}$.

The equalities (2.5) and (2.6) jointly with (5.2) mean that the reflected wave corresponds to the zero η_0 :

$$H_{\eta_0} = \frac{\eta_1^2 - \eta_0 \mu}{z_0(\mu - \eta_0)} \exp \left[i \left(\frac{kx}{\varepsilon} - \omega t \right) \right], \quad e_{\eta_0} = \exp \left[i \left(\frac{kx}{\varepsilon} - \omega t \right) \right],$$

and the wave incoming to the plasma boundary corresponds to the symmetric zero $-\eta_0$:

$$H_{-\eta_0} = \frac{\eta_1^2 + \eta_0 \mu}{z_0(\mu + \eta_0)} \exp \left[-i \left(\frac{kx}{\varepsilon} + \omega t \right) \right], \quad e_{-\eta_0} = \exp \left[-i \left(\frac{kx}{\varepsilon} + \omega t \right) \right],$$

6. EXPANSION IN THE TERMS OF EIGENFUNCTIONS

In the work [11] it was shown that from the non-flowing condition (2.11) and the condition on the electric field (2.10) it results that the trivial (equal to zero) solution of the present problem corresponds to the point $\eta_i = \infty$.

We will show that the system of equations (2.8) and (2.9) with the boundary conditions (3.1), (3.5) and (2.10) has the solution which can be represented

as an expansion by the eigenfunctions of the characteristic system:

$$h(x, \mu) = \frac{E_2}{z_0} \frac{\eta_0 \mu - \eta_1^2}{\eta_0 - \mu} \exp(i \frac{kx}{\varepsilon}) + \frac{E_1}{z_0} \frac{\eta_0 \mu + \eta_1^2}{\eta_0 + \mu} \exp(-i \frac{kx}{\varepsilon}) + \frac{1}{z_0} \int_0^1 \exp\left(-z_0 \frac{x}{\eta}\right) F(\eta, \mu) E(\eta) d\eta, \quad (6.1)$$

$$e(x) = E_2 \exp(i \frac{kx}{\varepsilon}) + E_1 \exp(-i \frac{kx}{\varepsilon}) + \int_0^1 \exp\left(-z_0 \frac{x}{\eta}\right) E(\eta) d\eta. \quad (6.2)$$

Here E_1 is given, and E_2 is unknown coefficient. Both of the variables (amplitudes of Debay) correspond to the discrete spectrum, $E(\eta)$ is unknown function, which is called eigenfunction of continuous spectrum.

Our purpose is to find the coefficient of the continuous spectrum and the relation which connects the coefficients of the discrete spectrum.

Let us substitute the expansion (6.1) into the boundary condition (3.1). We get the following equation in the interval $0 < \mu < 1$:

$$\int_0^1 \left[F(\eta, \mu) - F(\eta, -\mu) \right] E(\eta) d\eta + (E_1 + E_2) \varphi_0(\mu) = -\frac{2}{3} z_0 A_1 + z_0 A_1 \mu. \quad (6.3)$$

Here

$$\varphi_0(\mu) = \frac{\eta_1^2 - \eta_0 \mu}{\mu - \eta_0} + \frac{\eta_1^2 + \eta_0 \mu}{\mu + \eta_0}.$$

Extending the function $E(\eta)$ into the interval $(-1, 0)$ evenly we transform the equation (6.3) to the following form:

$$\int_{-1}^1 F(\eta, \mu) E(\eta) d\eta + (E_1 + E_2) \varphi_0(\mu) - z_0 A_1 \mu = -\frac{2}{3} z_0 A_1 \operatorname{sign} \mu. \quad (6.4)$$

Let us substitute the eigenfunctions of the continuous spectrum into the equation (6.4). We obtain singular integral equation with Cauchy kernel in the interval $(-1, 1)$:

$$(E_1 + E_2) \varphi_0(\mu) + \int_{-1}^1 \frac{\eta \mu - \eta_1^2}{\eta - \mu} E(\eta) d\eta - c \frac{\lambda(\mu)}{\mu} E(\mu) - z_0 A_1 \mu =$$

$$= -\frac{2}{3}z_0A_1 \operatorname{sign} \mu. \quad (6.5)$$

7. SOLUTION OF THE SINGULAR EQUATION

We introduce the auxiliary function

$$M(z) = \int_{-1}^1 \frac{\eta z - \eta_1^2}{\eta - z} E(\eta) d\eta, \quad (7.1)$$

the boundary values of which on the real axis above and below it are related by the Sokhotsky formulas:

$$M^+(\mu) - M^-(\mu) = 2\pi i(\mu^2 - \eta_1^2)E(\mu). \quad (7.2)$$

$$\frac{M^+(\mu) + M^-(\mu)}{2} = M(\mu), \quad -1 < \mu < +1, \quad (7.3)$$

where

$$M(\mu) = \int_{-1}^1 \frac{\eta \mu - \eta_1^2}{\eta - \mu} E(\eta) d\eta,$$

and the singular integral in this equality is understood as singular in the sense of principal value of Cauchy.

With the help of the Sokhotsky formulas for the dispersion and auxiliary function we reduce the equation (6.5) to the boundary condition of the problem of determination of analytic function by its jump on the contour:

$$\begin{aligned} \lambda^+(\mu)[M^+(\mu) + (E_1 + E_2)\varphi_0(\mu) - z_0A_1\mu] - \lambda^-(\mu)[M^-(\mu) + (E_1 + E_2)\varphi_0(\mu) - \\ - z_0A_1\mu] = \frac{i\pi}{3\eta_1^2}A_1\mu(\eta_1^2 - \mu) \operatorname{sign} \mu, \quad -1 < \mu < 1. \end{aligned}$$

This equation has general solution (see [15]):

$$\lambda(z)[\varphi(z)(E_1 + E_2) + M(z) - z_0A_1z] = \frac{2z_0A_1}{3c} \int_{-1}^1 \frac{\mu(\mu^2 - \eta_1^2) \operatorname{sign} \mu}{\mu - z} d\mu + C_1z,$$

where C_1 is an arbitrary constant.

Let us introduce auxiliary function

$$T(z) = \frac{1}{c} \int_{-1}^1 \frac{\mu(\mu^2 - \eta_1^2) \operatorname{sign} \mu}{\mu - z} d\mu.$$

Then from the general solution we can easy find $M(z)$:

$$M(z) = -(E_1 + E_2)\varphi(z) + z_0 A_1 z + \frac{2}{3} z_0 A_1 \frac{T(z)}{\lambda(z)} + \frac{C_1 z}{\lambda(z)}. \quad (7.4)$$

Let us eliminate the pole of the solution (7.4) in the infinity. We get that

$$C_1 = -z_0 A_1 \lambda_\infty.$$

Poles in the points $z = \pm \eta_0$ can be eliminated with the help of one equality since the functions constituting the general solution are uneven:

$$z_0 A_1 = \frac{(E_1 + E_2) \lambda'(\eta_0) (\eta_1^2 - \eta_0^2)}{(2/3)T(\eta_0) - \lambda_\infty \eta_0}. \quad (7.5)$$

We substitute the expansion (6.1) for the function $h(x, \mu)$ to the integral boundary condition (3.5). We get the following equation:

$$E_1 m(-\eta_0) + E_2 m(\eta_0) + \int_0^1 m(\eta) E(\eta) d\eta = -\frac{1}{36} z_0 A_1 \frac{1 - \alpha_p}{\alpha_p}. \quad (7.6)$$

In (7.6) the following designations were introduced:

$$m(\pm \eta_0) = \int_0^1 \left(\mu^2 - \frac{2}{3} \right) F(\pm \eta_0, \mu) d\mu, \quad m(\eta) = \int_0^1 \left(\mu^2 - \frac{2}{3} \right) F(\eta, \mu) d\mu.$$

The coefficient of the continuous spectrum we will find from the Sokhotsky formula (7.1) after the substitution of the general solution (7.4) into it:

$$E(\eta) = \frac{1}{2\pi i (\eta^2 - \eta_1^2)} \left[\frac{2}{3} \left[\frac{T^+(\eta)}{\lambda^+(\eta)} - \frac{T^-(\eta)}{\lambda^-(\eta)} \right] - \lambda_\infty \eta \left[\frac{1}{\lambda^+(\eta)} - \frac{1}{\lambda^-(\eta)} \right] \right]. \quad (7.7)$$

Let us notice that under the transition through the positive part of the cut $(0, 1)$ functions $T(z)$ and $\lambda(z)$ make jumps, which differ only by sign. Indeed, let us represent the formula for $T(z)$ in the following form:

$$T(z) = \frac{z}{c} \int_0^1 (\mu^2 - \eta_1^2) \left[\frac{1}{\mu - z} + \frac{1}{\mu + z} \right] d\mu.$$

This integral can be calculated easily in explicit form. On the cut this integral is calculated according to the following formula :

$$T(\eta) = \frac{\eta}{c} \left[1 + (\eta^2 - \eta_1^2) \ln \left(\frac{1}{\eta^2} - 1 \right) \right], \quad -1 < \eta < +1.$$

Now from the Sokhotsky formula for the difference of boundary values we obtain that under condition $0 < \eta < 1$ the following equality takes place:

$$\begin{aligned} \lambda^+(\eta) - \lambda^-(\eta) &= \lambda(\eta) \pm \frac{i\pi\eta(\eta_1^2 - \eta^2)}{c}, \\ T^+(\eta) - T^-(\eta) &= T(\eta) \pm \frac{i\pi\eta(\eta^2 - \eta_1^2)}{c}. \end{aligned}$$

Now one can naturally find that

$$\begin{aligned} T^+(\eta)\lambda^-(\eta) - T^-(\eta)\lambda^+(\eta) &= 2(T(\eta) + \lambda(\eta)) \cdot \frac{i\pi\eta(\eta^2 - \eta_1^2)}{c}, \\ \lambda^-(\eta) - \lambda^+(\eta) &= 2\frac{i\pi\eta(\eta^2 - \eta_1^2)}{c}. \end{aligned}$$

With the help of the last relations we find the coefficient of the continuous spectrum from (7.5):

$$E(\eta) = z_0 A_1 Q(\eta), \quad \text{where} \quad Q(\eta) = \frac{\frac{2}{3}\eta[T(\eta) + \lambda(\eta)] - \lambda_\infty \eta^2}{c\lambda^+(\eta)\lambda^-(\eta)}. \quad (7.8)$$

We introduce the integral

$$T_0(z) = \frac{1}{c} \int_0^1 \frac{\eta^2 - \eta_1^2}{\eta - z} d\eta.$$

It is evident that in the complex plane this integral is calculated by the following formula:

$$T_0(z) = \frac{z}{c} \left[\frac{1}{2} + z + (z^2 - \eta_1^2) \ln \left(\frac{1}{z} - 1 \right) \right].$$

With the help of this function we represent the dispersion function in the form: $\lambda(z) = 1 - zT_0(z) + zT_0(-z)$, the function $T(z)$ we also express in terms of this integral: $T(z) = zT_0(z) + zT_0(-z)$. The sum of two last expressions equals to: $\lambda(z) + T(z) = 1 + 2zT_0(-z)$. Let us note that the integral $T(-z)$ is

not singular on the cut $0 < \eta < 1$. The sum $\lambda(\eta) + T(\eta)$ on the cut $0 < \eta < 1$ is calculated in explicit form without applying to integrals:

$$\lambda(\eta) + T(\eta) = 1 + \frac{1}{c} \left[\eta - 2\eta^2 + 2\eta(\eta^2 - \eta_1^2) \ln(1/\eta + 1) \right].$$

We calculate the integrals $m(\pm\eta_0)$ and $m(\eta)$ in explicit form. The integrals $m(\pm\eta_0)$ can be determined easily:

$$m(\pm\eta_0) = -(\eta_0^2 - \eta_1^2) \left[-\frac{1}{6} + \eta_0 + \eta_0(\eta_0 - \frac{2}{3}) \ln(\frac{1}{\eta_0} - 1) \right].$$

Let us find the integral $m(\eta)$. We have:

$$m(\eta) = \int_0^1 \left(\mu^2 - \frac{2}{3}\mu \right) (\eta_1^2 - \eta\mu) \frac{d\mu}{\mu - \eta} - 2\eta_1^2 z_0 \left(\eta - \frac{2}{3} \right) \lambda(\eta) \theta_+(\eta).$$

Here $\theta_+(\eta)$ is the characteristic function of the interval $0 < \eta < 1$, i.e.

$$\theta_+(\eta) = \begin{cases} 1, & 0 < \eta < 1, \\ 0, & -1 < \eta < 0. \end{cases}$$

Computating the integral in the preceding equality we obtain that the integral $m(\eta)$ is calculated by the formula:

$$m(\eta) = \left(\frac{1}{6} - \eta \right) (\eta^2 - \eta_1^2) + \left(\eta - \frac{2}{3} \right) \left[-c + 2\eta^2 - \eta(\eta^2 - \eta_1^2) f_+(\eta) \right],$$

where

$$f_+(\eta) = \begin{cases} \ln \frac{1+\eta}{\eta}, & 0 < \eta < 1, \\ \ln \frac{1-\eta}{\eta}, & -1 < \eta < 0. \end{cases}$$

Now the equation (7.6) with the help (7.8) we rewrite in the form:

$$E_1 m(-\eta_0) + E_2 m(\eta_0) = -z_0 A_1 \left(\frac{1 - \alpha_p}{36\alpha_p} + Q_m \right), \quad (7.9)$$

where

$$Q_m = \int_0^1 m(\eta) Q(\eta) d\eta.$$

After that substituting the variable $z_0 A_1$ into the equation (7.9) according to (7.5), we derive the equation:

$$E_1 m(-\eta_0) + E_2 m(\eta_0) = - \left(\frac{1 - \alpha_p}{36\alpha_p} + Q_m \right) \frac{E_1 + E_2}{\frac{2}{3}T(\eta_0) - \lambda_\infty \eta_0},$$

from which we can find the amplitude required E_2 :

$$E_2 = -\frac{\alpha_p m(-\eta_0)A(\eta_0) + B(\eta_0)C(\alpha_p)}{\alpha_p m(\eta_0)A(\eta_0) + B(\eta_0)C(\alpha_p)} E_1. \quad (7.10)$$

We introduced the following designations in the formula (7.10):

$$A(\eta_0) = \frac{2}{3}T(\eta_0) - \lambda_\infty \eta_0, \quad C(\alpha_p) = \frac{1 - \alpha_p}{36} + \alpha_p Q_m,$$

$$B(\eta_0) = (\eta_1^2 - \eta_0^2)\lambda'(\eta_0).$$

Thus, all the coefficients of the expansions (6.1) and (6.2) are determined unambiguously, and this completes the proof of these expansions.

It is seen from the equality (7.10) that under condition $\alpha_p = 0$ we have: $E_2 = -E_1$, i.e. under condition of pure specular reflection of electrons from the boundary the wave reflectance equals to unity: $R = 1$, and the phase shift of the incoming and reflected waves is equal to 180° , i.e. $\phi = \pi$, from where we see that $\arg E_2 = \arg E_1 + \pi$.

Let us represent the formula (7.10) in the form which is more convenient for numerical analysis. Let us designate the ratio of amplitudes as K , $K = E_2/E_1$, the

$$K = -1 + \frac{\alpha_p [m(\eta_0) - m(-\eta_0)]}{\alpha_p m(\eta_0) + C(\alpha_p)D(\eta_0)}, \quad (7.9)$$

where

$$D(\eta_0) = \frac{B(\eta_0)}{A(\eta_0)} = \frac{(\eta_1^2 - \eta_0^2)\lambda'(\eta_0)}{(2/3)T(\eta_0) - \lambda_\infty \eta_0}.$$

8. LONG WAVE LIMIT

For study of the incoming wave reflectance $R = |K|^2$ and the phase shift $\phi = \arg K$ we will use the formula (7.11).

We consider the dispersion equation with small values of the wave number k :

$$\lambda\left(i\frac{\varepsilon z_0}{k}\right) = \lambda_\infty - \frac{\lambda_2 k^2}{\varepsilon^2 z_0^2} = 0. \quad (8.1)$$

We assume the frequency ω is complex: $\omega = \omega_0 + i\omega_1$. Then the quantity γ is complex also: $\gamma = \gamma_0 + i\gamma_1$. Here $\gamma_0 = \omega_0/\omega_p - 1$, $\gamma_1 = \omega_1/\omega_p$. From

the equation (8.1) we find that when k is small $\gamma_0 = 0.3k^2$, $\gamma_1 = -0.5\varepsilon$. We express the parameters of the problem in terms of k and ε : $\lambda_\infty = 0.6k^2(1-i\varepsilon)$,

$$z_0 = \frac{1}{2} - \frac{1 + 0.3k^2}{\varepsilon}, \quad \eta_1^2 = -i\frac{\varepsilon}{3}(1 + 0.3k^2), \quad \eta_0 = \frac{1 + 0.3k^2 + i0.5\varepsilon}{k}.$$

With the help of these parameters let us carry out the study of the reflectance and the phase shift in long wave limit (when k is small by magnitude).

On the Fig. 3 one can see the dependence of the reflectance R on the wave number k for the case when $\varepsilon = 10^{-2}$. The curves 1, 2, 3 correspond to the following values of the accommodation coefficient $\alpha_p = 0.1, 0.5, 1.0$. The curve 4 corresponds to the diffuse boundary conditions (see [10]). From the graph it is seen that when k takes on small values the curve 3 (corresponding to $\alpha_p = 1$) coincides practically with the curve 4 (to which corresponds the case $q = 1$), which was obtained by the linearization of the reflectance value by k . Taking into account that under $\alpha_p = 0$ the reflectance is equal to reflectance for specular boundary conditions (i.e. when $q = 0$) we can conclude that specular accommodative boundary conditions approximate specular and diffuse boundary conditions under $\alpha_p = q$ very well.

On the Fig. 4 the dependence of the reflectance R on the quantity ε for the case $\alpha_p = 1$ is represented. The curves 1, 2, 3, 4, 5 correspond to the following values of the wave number $k = 0.001, 0.01, 0.05, 0.1, 0.2$. The more accurate analysis shows that with the growth of the accommodation coefficient the value of the reflectance decreases.

On the Fig. 5 the dependence of the reflectance R on the value of the accommodation coefficient α_p for the case $\varepsilon = 10^{-3}$ is presented. The curves 1, 2, 3, 4 correspond to the following values of the wave number $k = 0.05, 0.1, 0.15, 0.25$.

On the Fig. 6 the dependence of the angle $\phi = \arg K$ (of the phase shift) on the quantity ε for the case $k = 0.2$ is represented. The curves 1, 2, 3 correspond to the following values of the accommodation coefficient $\alpha_p = 0.1, 0.5, 1$. The analysis shows that the dependence between the values of the angle ϕ and the wave number and the accommodation coefficient as

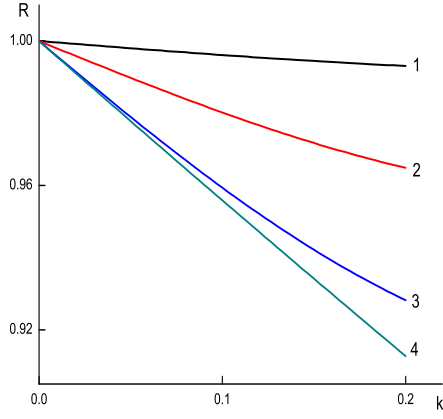


Fig. 3.

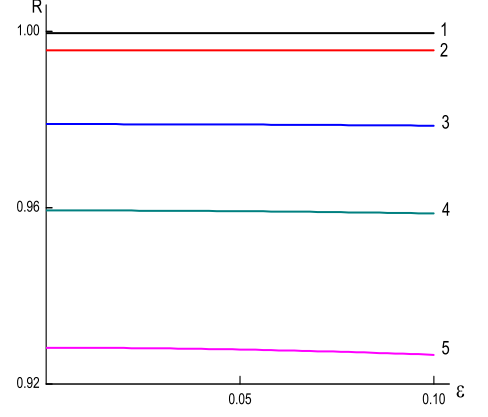


Fig. 4.

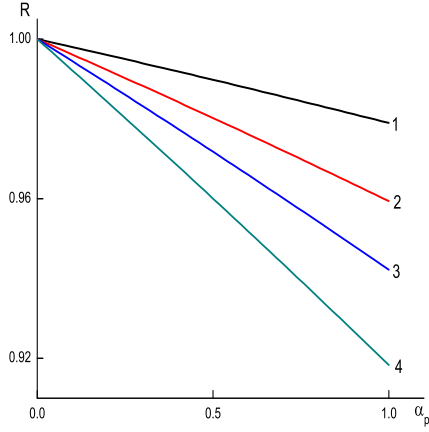


Fig. 5.

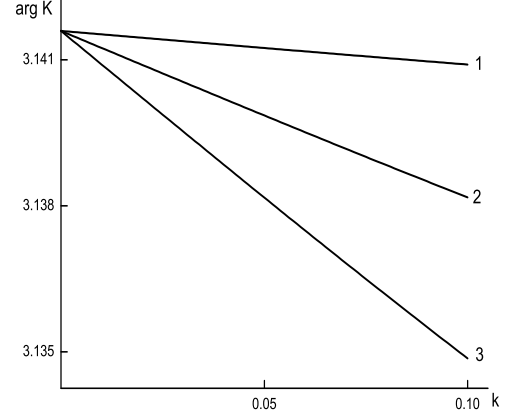


Fig. 6.

well is small.

9. CONCLUSION

In the present work new boundary conditions for the questions of plasma wave reflection from the plane boundary of a half-space of degenerate plasma were proposed. These boundary conditions are naturally called as specular accommodative conditions. Such boundary conditions are most adequate for the problems of normal propagation of plasma waves (perpendicular to the boundary), since accommodation coefficient under such boundary conditions is normal electron momentum accommodation coefficient.

In the present paper the analytical solution of the problem of plasma wave reflection from a boundary with normal electron momentum accommodation is obtained. The analysis of the main parameters of the problem in long wave limit is carried out. This analysis shows that the boundary conditions proposed are intermediate between pure specular and pure diffuse boundary conditions. Indeed, from the Fig. 3 it is seen that all the graphs showing the dependence between the reflectance and the wave number are located between graphs corresponding to specular and diffuse boundary conditions.

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